Serie 4 on Chapter 3: Impedance matching of the internal impedance of the voltage source to the impedance of the load

The frequency of the sinusoidal voltage source is equal to 100 MHz.

The internal impedance of the sinusoidal voltage source is modeled by a resistor of 100 Ohms in series with an inductor of 600 nH.

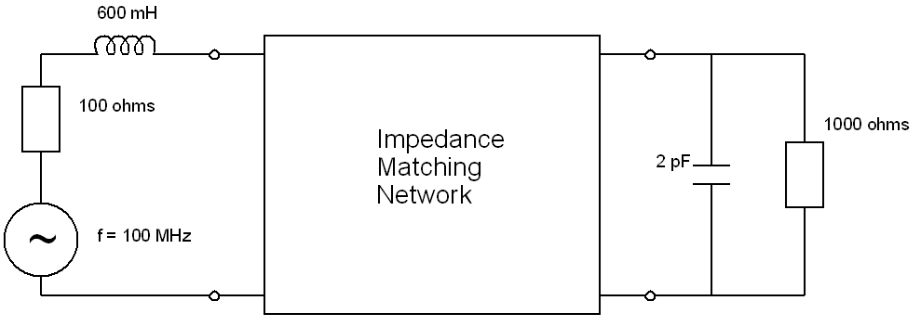
The impedance of the load is modeled by a capacitor of 2 pF in parallel with a resistor of 1’000 Ohms.

1) The number of components of the impedance matching network is equal to 2. Calculate the values of the passive components of the impedance matching network by using the two methods explained in chapter 3.

2) What type of impedance matching network do we obtain (low-pass filter, band-pass filter, high-pass filter) ? Thanks to justify your answer.

Answer to Question 1:

We have to design the impedance matching network for the circuit shown below:



For this purpose, we have two methods that are described in the course (page 3-9):

Absorption:

The reactances of the source and of the load can be taken into account in the impedance matching network by placing the components such that the functional capacitor of the matching network is in parallel with the capacitor of the load and the functional inductor of the matching network is in series with the inductor of the voltage source.

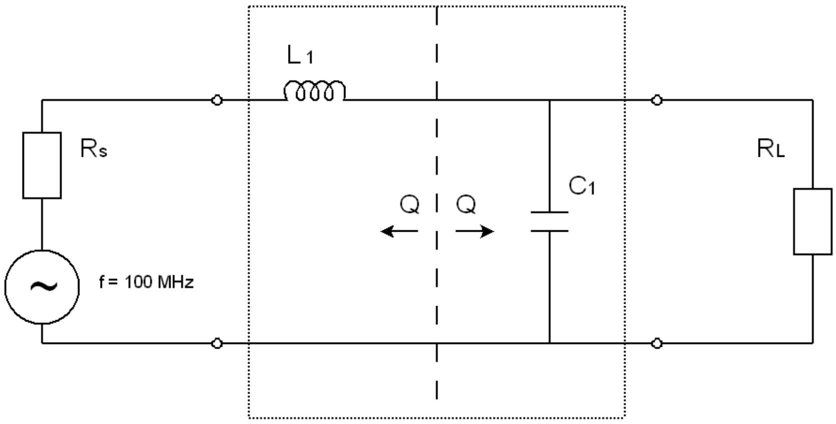
Resonance:

Cancel out the effect of the reactances of the source and of the load by placing a reactance of the opposite sign in parallel or in series.

In order to have a complete treatment of the exercise, we will use both methods and we will check that we obtain the same topology of impedance matching network as well as the same values of components.

First method: absorption

The idea of absorption is to use the already present capacitive and inductive elements of the source and of the load and to add only the missing quantity to these in order to obtain the elements of the matching network. For this reason, we will consider first the circuit without any reactive elements at source and load and calculate the required inductor and capacitor for having the matching. Another way to see this is to consider that reactive elements are all included in the matching network itself, as shown below.



The quality factor has to remain the same on both sides of the network. We have a series circuit on the left and a shunt circuit on the right, so we can write:

$$R_s = \frac{R_L}{1+Q^2} \Leftrightarrow Q = \sqrt{\frac{R_L}{R_s} - 1} = \sqrt{\frac{1000}{100} - 1} = \sqrt{9} = 3$$

By definition of the quality factor of a capacitive or inductive circuit, we have:

$$Q = \frac{\omega_0 L_1}{R_s} \Leftrightarrow L_1 = \frac{R_s Q}{\omega_0}$$

$$Q = R_L \cdot \omega_0 C_1 \Leftrightarrow C_1 = \frac{Q}{R_L \omega_0}$$

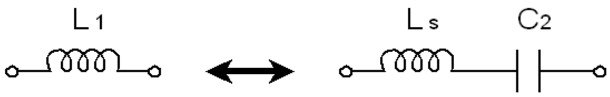
In our case, we have:

$$L_1 = \frac{R_s Q}{\omega_0} = \frac{100 \cdot 3}{2\pi 10^8} H = 477.46\, nH$$

$$C_1 = \frac{Q}{R_L \omega_0} = \frac{3}{10^3 \cdot 2\pi 10^8} = 4.7746\, pF$$

As explained in the course, we should normally add an inductance and a capacitance to the elements already present at source and load in order to reach the above calculated values. However, we have a problem because L_s is larger than L_1. How can it be solved ?

The solution is to add a capacitor in series with the inductor in order to decrease the reactive part of the source impedance:



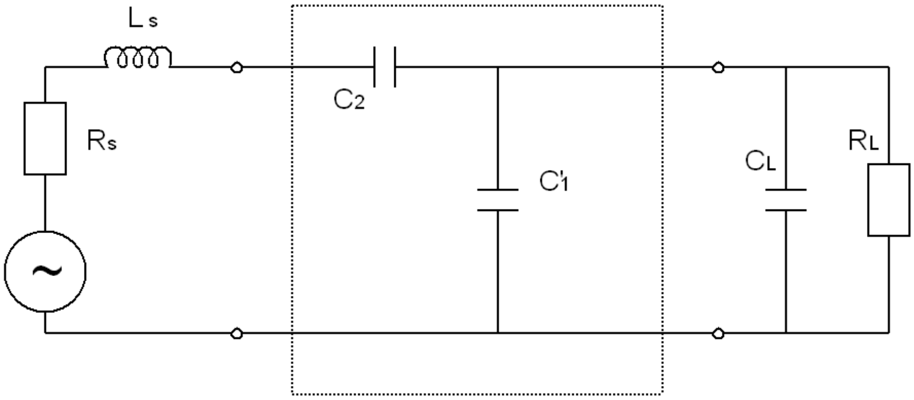
The value of this capacitor is calculated as follows:

$$j\omega_0 L_1 = j\omega_0 L_s + \frac{1}{j\omega_0 C_2} \Leftrightarrow C_2 = \frac{1}{\omega_0^2 (L_s - L_1)}$$

In our case, we have:

$$C_2 = \frac{1}{\omega_0^2 (L_s - L_1)} = \frac{1}{4\pi^2 10^{16} \cdot (600 \cdot 10^{-9} - 477.46 \cdot 10^{-9})} = 20.671\ pF$$

This gives the following impedance matching circuit:



where :

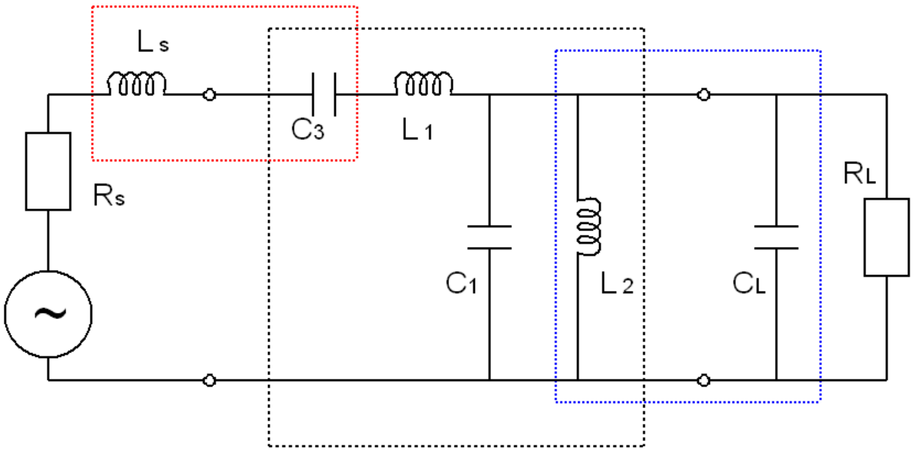
$$C_2 = 20.671\ pF$$

$$C'_1 = C_1 - C_L = 4.7746\ pF - 2\ pF = 2.7746\ pF$$

Second method: resonance

The idea of resonance is to add the capacitor C_3 in series with L_s(red box) and the inductor L_2in parallel with C_L(blue box) in order to cancel the imaginary (reactive) part of source and load circuits as shown below.

When it is done, only the standard impedance matching cell L_1 and C_1 is required for matching the remaining real part as done previously with the absorption method. Of course, this method works only at one frequency (here ω_0).

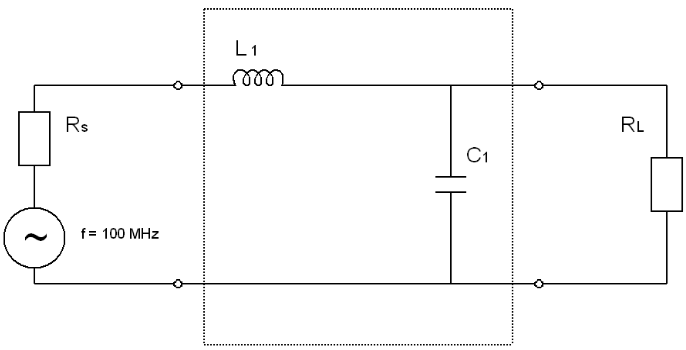


At resonance, the values of C_3 and $L_2$are given as follows:

$$L_s C_3 = \frac{1}{\omega_0^2} \Leftrightarrow C_3 = \frac{1}{\omega_0^2 L_s} = \frac{1}{(2\pi 10^8)^2 \cdot (600 \cdot 10^{-9})} = 4.2217 \text{ p}F$$

$$L_2 C_L = \frac{1}{\omega_0^2} \Leftrightarrow L_2 = \frac{1}{\omega_0^2 C_L} = \frac{1}{(2\pi 10^8)^2 \cdot (2 \cdot 10^{-12})} = 1.2665 \ \mu H$$

At ω_0, the circuit becomes thus as follows:



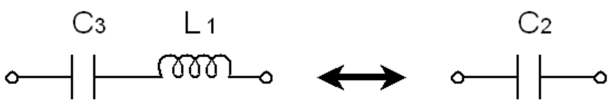
This is exactly the same circuit that we obtained with the absorption method and $L_1$and $C_1$are calculated in the same manner:

$L_1 = 477.46\, nH$

$C_1 = 4.7746\, pF$

The exercise in not finished since we want to use two elements to achieve impedance matching.

First, we want to replace $C_3$and $L_1$by one component, but we do not know if it is a capacitor or an inductor. By calculating their reactances, we see that a capacitive behaviour dominates. We can do a transformation similar as the one done previously:



The value of the capacitor $C_2$is calculated as follows:

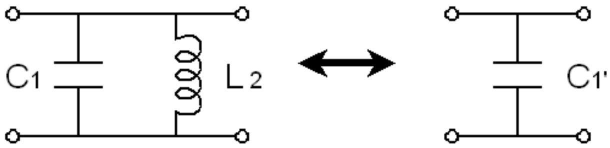
$$\frac{1}{j\omega_0 C_3} + j\omega_0 L_1 = \frac{1}{j\omega_0 C_2} \Leftrightarrow \frac{1}{C_3} - \omega_0^2 L_1 = \frac{1}{C_2}$$

So we have:

$$C_2 = \frac{1}{\frac{1}{C_3} - \omega_0^2 L_1} = \frac{C_3}{1 - \omega_0^2 L_1 C_3} = \frac{4.2217\, pF}{1 - (2\pi 10^8)^2 \cdot 477.46\, nH \cdot 4.2217\, pF} = 20.671\, pF$$

We check that it is the same value as calculated previously with the absorption method.

Similarly, we want to replace $C_1$and $L_2$by one component, but we do not know if it is a capacitor or an inductor. By calculating their susceptance, we see that a capacitive behaviour dominates. We thus have the following transformation:



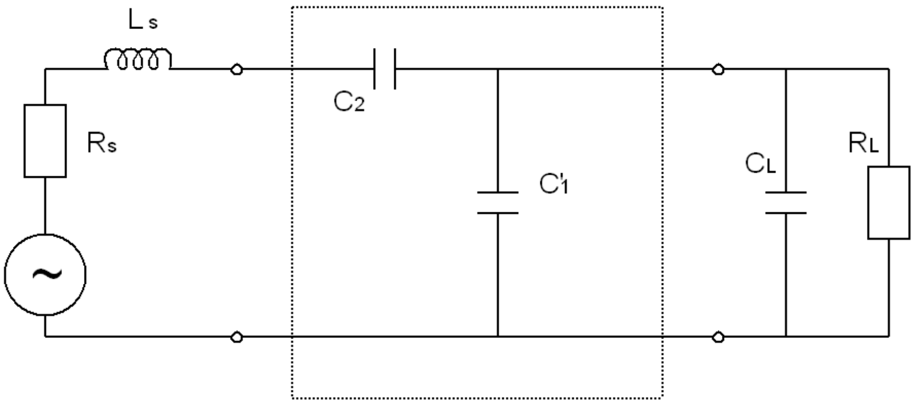
The value of the capacitor $C_{1'}$is calculated as follows:

$$j\omega_0 C_{1'} = j\omega_0 C_1 + \frac{1}{j\omega_0 L_2} \Leftrightarrow C_{1'} = C_1 - \frac{1}{\omega_0^2 L_2}$$

So we have:

$$C_{1'} = C_1 - \frac{1}{\omega_0^2 L_2} = 4.7746pF - \frac{1}{(2\pi 10^8)^2 \cdot 1.2665 \cdot 10^{-6}} = 2.7746\ pF$$

The impedance matching network finally reduces to the following circuit:



$$C_2 = 20.671\ pF$$

$$C'_1 = 2.7746\ pF$$

As expected, this is the same topology of matching network and the same values of elements as calculated previously with the absorption method.

Answer to Question 2:

What type of impedance matching network do we obtain (low-pass filter, band-pass filter, high-pass filter) ?

At $\omega_0 = 0$: the signal is blocked by C_2
At $\omega_0 = \infty$: the signal is shorted by $C_{1'}$.

Therefore, the impedance matching network corresponds to a band-pass filter.